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PRELIMINARY DESIGN AND ACCURACY ANALYSIS OF A GROUND-LAUNCHED MULTIPLE ROCKET SYSTEM FOR BREACHING MINE FIELDS

By

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Problem: The overall problem is to evaluate the effectiveness of breaching a mine field by using a trailer-load of rocket delivered weapons. This basic problem consists of three major parts; the warhead effectiveness and kill criteria of the mine field, the preliminary design and accuracy of the rocket system, and the system effectiveness of breaching a mine field using the results from the warhead effectiveness and system accuracy. This memo deals with the preliminary rocket system design and accuracy analysis part of the problem.

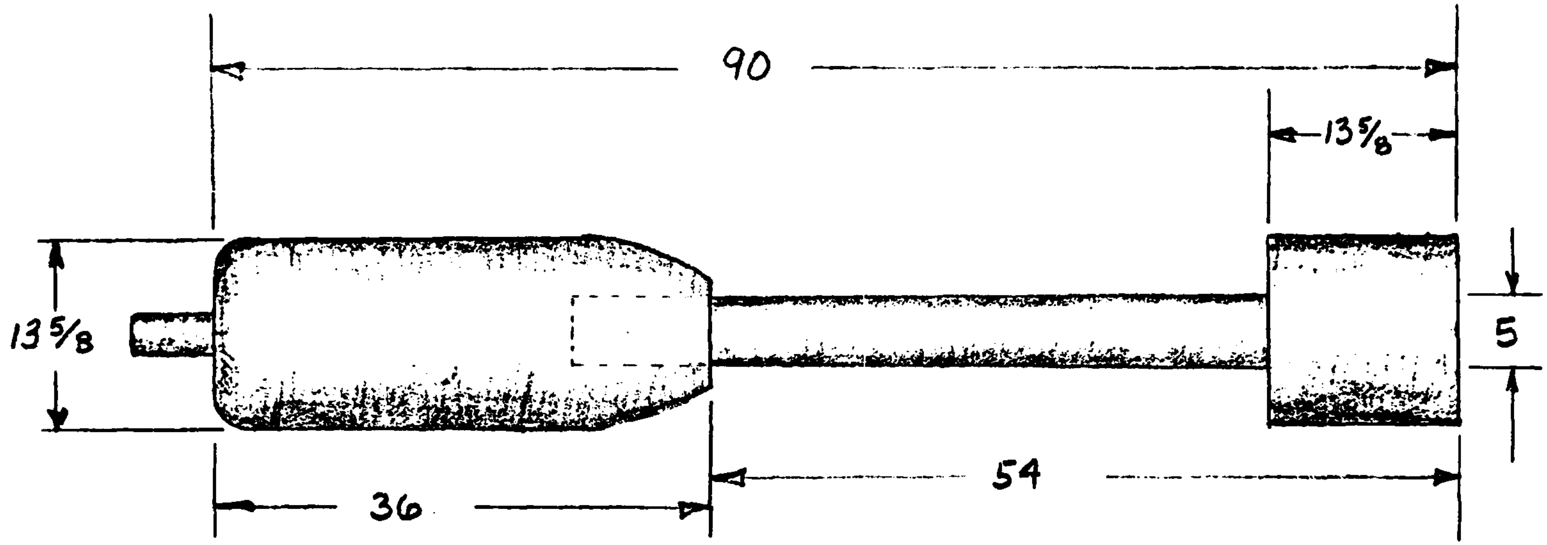
Purpose: The purpose of this report is to design and evaluate the accuracy of a proposed multiple rocket system. This implies a preliminary baseline design for which the accuracy is evaluated. Accuracy implies the basic deviations resulting from an attempt to place the rockets in a pattern required to breach the mine field.

Approach: The approach consists of tradeoffs between the rocket baseline design, the projectile trajectory, and the accuracy. The trend of all the tradeoffs is to make the rocket system more accurate. The potential of the system to effectively breach a mine field is greatly influenced by the system accuracy. Therefore, considerable effort is made to make the accuracy analysis credible. Toward this end the baseline rocket design is greatly influenced by the wealth of data obtained in similar World War II designs.

Baseline Design: The major contributors to the baseline design are the required system components, the performance, and similarity. The warhead and fuzes are

required components in the baseline design. Essentially, the warhead consists of a cylinder 13 5/8 inches in diameter, approximately 24 inches long (including the parachute). The warhead weighs approximately 150 pounds. The baseline vehicle maximum diameter is set at 13 5/8 inches. The required range performance entered the baseline design only as an input on the requirements of the rocket motor. A maximum range of about one thousand yards was requested. This requires a burnout velocity around 400 ft/sec (an easy requirement). As indicated later in this report, keeping the range short improves the accuracy. The greatest influence on the baseline rocket design is similarity. The baseline rocket system is modeled after designs used and tested in the 1940's. The design choice was dictated by the availability of theory and data as well as the fact that the design is simple and meets all performance criteria.

The baseline design is shown in Fig. 1. The figure also includes a list of the estimated physical properties and performance data. Essentially the baseline round consists of the head coupled to a smaller rocket motor with a ring tail. The head and the ring tail are of the same diameter. This design is compatible with tube, rail, or rack launch. Aerodynamically, the design has a relatively high drag coefficient. However, drag is of secondary importance at the low velocities. The warhead is rounded at the front and boattailed at the rear. The boattail forestalls separation so the flow will pass through the ring tail. The long motor length increases the static stability and improves the round accuracy by increasing the transverse radius of gyration. This latter effect is evaluated in Appendix B.



<u>BASELINE PHYSICAL AND PERFORMANCE FACTORS</u>

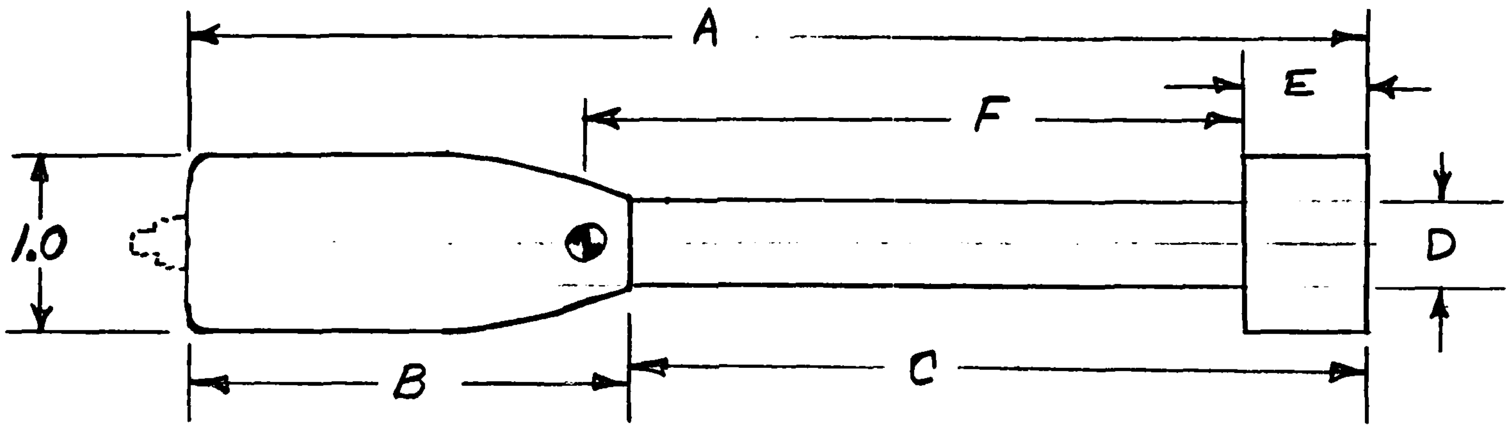
Center of Gravity (cg)	26 in (from head)	Initial Weight (W_i)	210 lbs.
Transverse Radius of Gyration (K)	1.88 calibers	Propellant Weight (Wp)	12.5 lbs.
Yaw Oscillation Distance (λ)	280 feet	Specific Impulse (Isp)	210 sec.
Static Margin (Xcp - Xcg)	2 calibers	Burntime (t_b)	0.5 sec.
Thrust (T)	5250 lbs.	Acceleration (G)	25 g's.

FIG. 1 -- The Baseline Design Including the Basic Physical and Performance Factors

The total baseline vehicle weighs 210 pounds. The rocket motor has a delivered specific impulse (Isp) of 210 seconds. This results in a propellant weight of 12 ½ pounds to yield a 400 ft/sec burnout velocity. The baseline rocket burntime is ½ second. The rocket average thrust is 5,250 pounds (about 25 g's). The short burnout and high acceleration are required for rocket accuracy (Appendix B).

During the Second World War the California Institute of Technology designed, developed and tested a large number of barrage rockets. Some of which like the 4.5 inch diameter Mk1 were mass produced and used by both the Army and the Navy. The ballistic data of these rocket tests is summarized in reference 1. The design similarity of this baseline vehicle to many of these rockets is shown in Fig. 2. The rockets shown vary in diameter from 2.5 inches to 7.2 inches. The baseline round is somewhat larger at 13 5/8 inches is diameter. In Fig. 2 the burnout velocity of the rockets range between 300 and 400 ft/sec. The table in Fig. 2 lists the various rocket design dimensions. The vehicle similarity becomes quite apparent when their dimensions are compared in calibers. One caliber equals the diameter. The vehicle sketch in Fig. 2 showns the 7.2 inch diameter Mk12. The other vehicles listed in the table differ a little from the 7.2 inch Mk12 in the nose and boattail shape. All the vehicles have ring tails with diameters equal to that of the heads. Figure 2 indicates a close similarity in the design of the baseline vehicle and the 7.2 inch diameter Mk12.

<u>Trajectories:</u> The baseline trajectory is shown in Fig. 3. The initial launch



VEHICLE	BURNOUT VELOCITY	A calibers	B calibers	C calibers	D calibers	E calibers	F calibers
7.2" dia. Mk.12	400 ft/sec	6.6	2.6	4.0	0.45	~0.7	3.8
4.5" dia. Mk.1	355	6.4	2.9	3.5	0.50	~1.0	3.3
2.5" dia. Grenade	350	7.9	3.0	4.9	0.50	~1.0	4.0
4.5" dia. (1000 yd)	335	7.4	2.9	4.5	0.50	~1.0	3.9
13⅝" dia. (BASELINE)	400	6.6	2.6	4.0	0.38	1.0	3.4

FIG. 2 -- Design Similarity of the Baseline Vehicle to Existing Designs of Various Diameters

angle (QE_i) is 30^o with a ½ second rocket burntime. Projectile burnout occurs 90 feet down range. The solid line in Fig. 3 represents the trajectory of the rocket system without the deployment of the parachute. The numbers represent the time after launch in seconds. Four examples of the trajectory variation caused by chute deployment are also shown. The solid triangles indicate the start of parachute deployment. The dashed lines continue the trajectory under the high parachute drag to impact. The impact angle is also presented. The first trajectory deploys the chute at burnout and represents the shortest range. Parachute deployment before burnout is not considered due to motor separation problems. Deployment of the chute at 8 sec (the last dotted line) represents essentially the maximum range.

The warhead fuzing requires a time of about 5 sec from chute deployment to impact. A ground impact angle greater than 65^o aids in warhead effectiveness. These requirements are met by the baseline trajectory. The effects of the trajectory and variations on the accuracy of the system are discussed in Appendix A.

Accuracy: The system accuracy indicates the capability of placing one round relative to the preceeding rounds. This dispersion type of accuracy is sufficient for breaching mine fields. The round-to-round type of accuracy is much less demanding of the system than the accuracy necessary to hit a specific target. For example, a constant cross wind affects all of the rounds the same and therefore is not a source of error. Variations in cross winds (gusts) between rounds do affect the system accuracy.

The accuracy analysis defines five major sources of system error. These consist

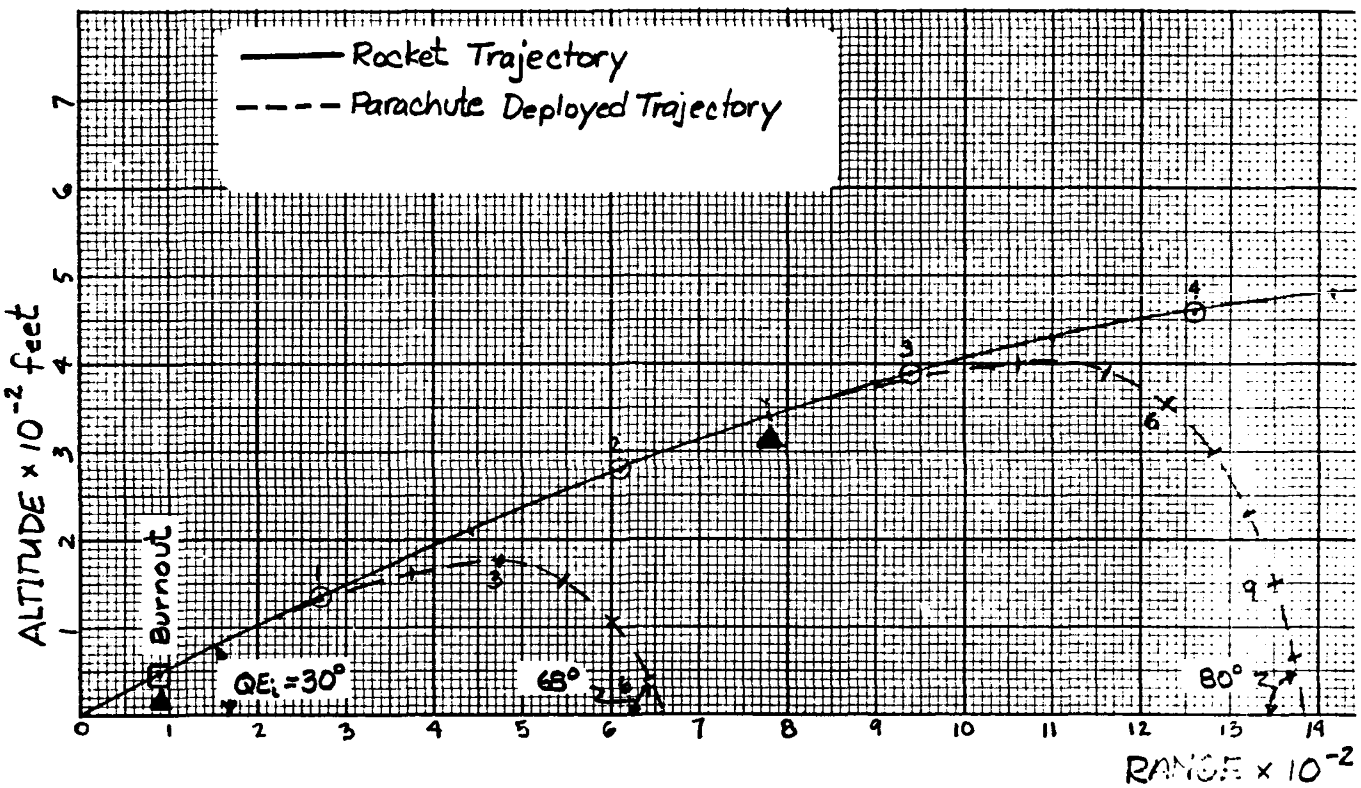
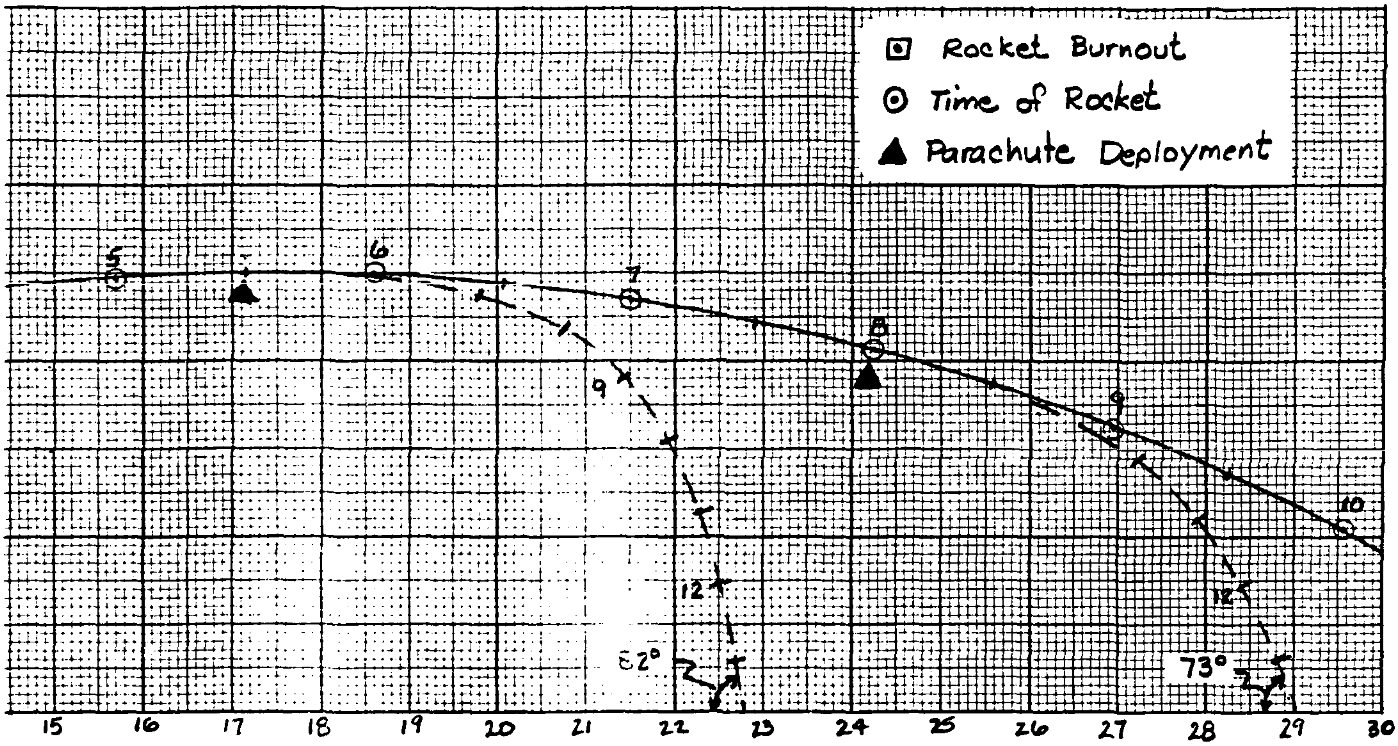


FIG. 3 -- Baseline Trajectory



Rocket Burnout
Time of Rocket
Parachute Deployment
5
6
7
8
9
10
9
12
12
82°
73°
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30

feet

of the errors associated with the launcher orientation, the rocket thrust direction and impulse, the fuze variation in timing, the chute opening deviations, and the drift from wind gusts. Each of these error sources are separately analyzed to evaluate the error they cause in both the deflection and range directions.

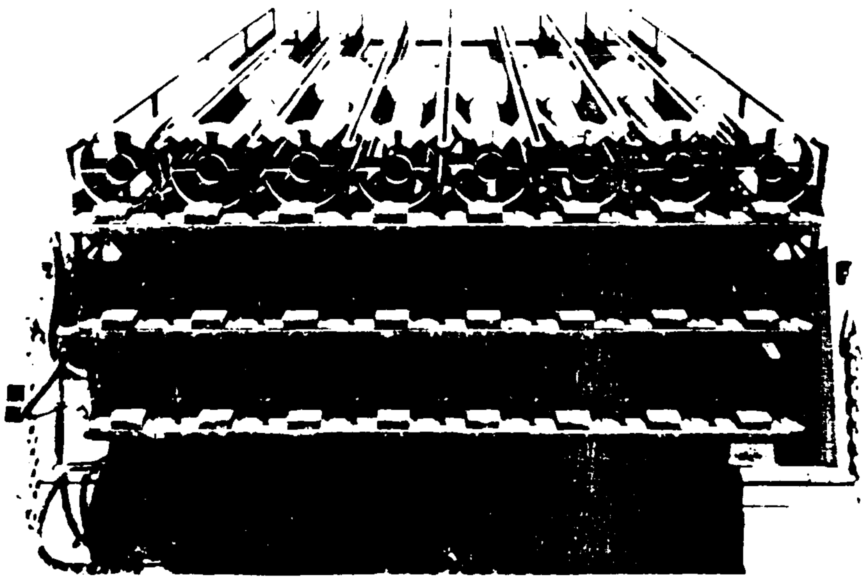
Launcher Errors: The launcher supports and directs the rockets before and during firing. Launchers come in various sizes and types as indicated in Ref. 2. "Rocket Launchers for Surface Use" (Ref. 2) shows the design and type of launchers developed for use during the Second World War. For multiple rocket (barrage type) application there has been little work performed in launcher theory, design, or development since the Second World War.

A few basic requirements influence the launcher design. 1) The rocket diameter is 13 5/8 inches. 2) The launcher system must hold 20-50 rockets (the actual number depends on the total system weight and the number of rounds necessary to breach a given length of mine field). 3) The launcher and rocket system must be mobile (e.g., pulled as a trailer behind a tank). 4) The launcher must be rigid and resistant to changes in attitude during launch. Transient motion after the round leaves the launcher is acceptable. However, permanent elevation or asimuth changes in orientation after launch must be minimized. 5) An effective launcher length of about 5 feet is needed for accuracy. The effective launcher length represents the distance the round travels while constrained to motion in one direction. The five foot distance is measured from the rear of the head (at 13 5/8 inch diameter) to the front of the launcher. This results in an overall launcher length of about 11 feet.

The first three requirements result from existing hardware (rocket diameter) or operational considerations. Requirements 4 and 5 result from accuracy considerations. The launcher motion requirement 4 is compounded by the necessity of having a high thrust, short burn rocket motor for rocket accuracy (Appendix B). The greatest amount of launcher motion results from the impact of the rocket exhaust on blunt launcher surfaces. The launcher design should minimize frontal projected areas on which the gas can impinge. The use of rails instead of tubes is preferred since relative close tolerance between rocket and tube presents a large area for recoil from the exhaust of a preceeding round. However, if very high thrust levels are required where the exhaust gas expands to diameters greater than 14 inches in less than 5 feet, a tube launcher would be preferred. At these high thrust levels the expanding gas (in a rail launcher) could impact the heads of the other rounds causing launcher motion before the vehicle has cleared the launcher. This motion during launch must be avoided (requirement 4) as it can greatly magnify the system errors.

Figure 4 shows an example of a multiple rail launcher taken from Ref. 2. This particular launcher mounts on the bed of a 6 x 6 truck and carries 24 7.2 inch diameter rounds. The Army developed a similar launching system having 48 rails (T-96). The launcher system required in this study would be considerably larger since the baseline round has a 13 5/8 inch diameter. The baseline round weighs about four times that of the 7.2 inch diameter rounds shown in Fig. 4. However, Fig. 4 represents an example of a multiple rail launcher which fires

7.2-in. Type 2 launcher on 2½-ton, 6x6 truck.



Rear view of 7.2-in. Type 2 launcher

FIG. 4 -- A 24 Rail Type 2 Launcher for 7.2 Diameter Rockets

rounds of similar geometry to the baseline rocket.

Launcher-induced errors are assumed to result from two major causes. The first source of error is the variation in rail direction. This implies that one launch rail system is not oriented in the same direction as the next. The cause of this error is in manufacturing tolerance and the construction of the launcher. The launcher is usually composed of subassemblies to improve handling. This rail alignment error can be kept small. The standard deviation around the mean launch angle is estimated at 3 mils (milli-radians). The second cause of launcher error is the change of position of the entire launcher caused by rocket fire. To keep this error small requires a relatively massive launcher with a minimum of projected area on which the exhaust gases can impinge. The critical time of launch is after the rocket has left the launcher and the exhaust gases then impinge on the launcher itself. An error of 8 mils is assumed to result from this launcher movement. Both launcher standard deviations are assumed randomly oriented about a mean launch orientation.

These launcher-induced deviations result in a similar type of error. This error being represented by an angular deviation in the mean launch direction. Summing these two errors (root mean squared) and correcting for randomness ($2/\pi$) gives a launch angle deviation in both the range and deflection direction of 5.4 mils. A 5.4 mil error in deflection (to the side) results in the rocket straying to the side 5.4 feet for every 1000 feet of distance. A 5.4 mil error in the range or pitch orientation of the launcher results in a quite different effect on the system error. As discussed in Appendix A this particular type of trajectory tends to decrease the

amount of error in the range direction caused by pitch angle deviations. The range and deflection errors on the rocket system caused by the launcher deviations discussed above are

$$\sigma_{\ell D} = 5.4 \text{ mils} = 5.4 \text{ (R/1000)ft.} \qquad (1a)$$

$$\sigma_{\ell R} = 5.4/2 \left(\frac{R-570}{R}\right) \text{mils} = 2.7 \left(\frac{R-570}{1000}\right) \text{ft} \qquad (1b)$$

where ℓ represents the launcher and the D and R represent the deflection and range directions. The term R/1000 converts mils to feet deviation at the range R (expressed in feet).

Rocket Errors: The rocket system causes the major source of system errors. Rockets and their accuracy were analyzed and tested by California Institute of Technology personnel during the Second World War. References 3 and 4 represent part of the results of their analyses. "Exterior Ballistics of Rockets" (reference 3) presents an analysis of rockets in a relatively rigorous manner. "Artillery and Aircraft Rockets" (reference 4) is a much less rigorous presentation with numerous examples taken from test data. An analytical evaluation of the rocket accuracy using these references in presented in Appendix B.

In summary, the major source of dispersion for the rockets result from thrust malalignment and propellant total impulse variations. Thrust malalignment is an off-axis component of the rocket thrust which does not pass through the

center of gravity of the round. This thrust malalignment produces a moment or turning of the vehicle around its center of gravity. The moment is relatively small as the malalignment is in the order of hundreds of an inch (e.g., 2/100). However, right at launch, as the rocket leaves the tube, the forward velocity is low. This low velocity produces small fin restoring moments. Initially, the perturbing moment caused by the thrust malalignment is greater than the restoring moment due to the fin aerodynamics. As the velocity of the rocket increases the fin restoring moment dominates the pitching moment and thrust malalignment is no longer a factor in the accuracy. However, the dominance of the thrust malalignment early in flight results in a net angular deviation from the intended direction of motion. This angle is random (has equal probability in any direction) and is usually expressed in mils. The analysis in Appendix B shows a rocket angular deviation dependence on the rocket burn distance. Reducing the burn distance reduces the dispersion. The rocket burn distance is proportional to the vacuum burnout velocity, (V_{bv}) and the rocket burntime (t_b), $d_b \approx \frac{1}{2} V_{bv} t_b$. Therefore to keep the burn distance short, the burntime was kept small (0.5 sec), and the rocket vacuum burnout velocity was kept to a minimum (400 ft/sec), consistent with meeting the maximum range requirements. Using these values and the assumptions made in Appendix B, the resulting standard deviation is 21 mils in both range and deflection.

Variation in the rocket propellant total impulse from round to round is another source of rocket error. This source causes the rocket vacuum burnout velocity to vary from round to round, resulting in a deviation in range. This variation

in rocket impulse causes no component of deflection error. The results of rocket tests (reference 4) indicated an impulse variation of 1% (mean deviation). This implies 1.25% standard deviation or 12 ½ mils.

Standard angular deviations caused by thrust malalignment is similar in nature to the angular deviation caused by the launcher. The range error contribution when coupled to the dynamics of the trajectory (Appendix A) result in a range error similar in form to that of the launcher. The rocket error contribution to the standard deviations in range and deflection are

$$\sigma_{rD} = 21 \frac{R}{1000} \text{ ft} \qquad (2a)$$

$$\sigma_{rR} = \left\{ \left[\tfrac{1}{2}(21) \right]^2 + (12.5)^2 \right\}^{\frac{1}{2}} \frac{R-570}{1000} \text{ ft} \qquad (2b)$$

where the lower case r represents the rocket. In Eq. 2b the first term (21/2) represents the range error due to thrust malalignment and the second term (12.5) that is caused by variations in the rocket propellant impulse.

Fuze Errors: The fuze initiates the deployment of the chute after a given time interval. This time interval is set in the fuze, and controls the range of the rocket system. As noted on the baseline trajectory (Fig. 3) the added range after chute deployment is essentially a constant value (about 570 feet). The range is therefore controlled by the time before chute deployment. Variations in fuze timing will therefore cause a deviation in range (range error). There

is no lateral or deflection error contributed by a variation in fuze timing.

The fuze, in order to time, must have a starting reference. The firing pulse is not a good reference due to the variation in propellant ignition delay times. This time variation in igniting the propellant grain should be eliminated from the fuze timing. This can be done by starting the timing after the rocket has moved a given distance down the launch rail. With the starting time fixed, a timer is set in the fuze to deploy the chute.

The fuze error results from a deviation in the fuze time setting. For example, if the rocket range component of velocity is 300 feet/sec and the fuze timing variation is ± 1/10 of a second, the error contribution in range is ± 30 feet. The fuze timing tradeoff is cost versus larger range deviations for lower accuracy timing. The accuracy must be viewed for the overall system, and not just the fuze contribution. A standard deviation in fuze time of 50 milli-seconds (1/20 second) is used as a compromise. The standard deviations in deflection and range caused by the fuze timing variations are

$$\sigma_{fD} = 0 \qquad (3a)$$

$$\sigma_{fR} = (1/20)\ (360) = 18 \text{ ft} \qquad (3b)$$

where the subscript f represents the fuze. The range component of velocity is 360 ft/sec. Designing the system for greater ranges requires higher average

velocities which directly affect the error source caused by fuze time variations.

Chute Deployment Errors: The parachute begins deployment following the function of the fuze. An explosive charge accelerates the rocket motor backwards pulling out the parachute. The chute deployment phase of flight is the distance traveled by the warhead from the fuze function to the point at which the parachute is fully open. Variations in this deployment distance represent a source of error in range.

Data for the deployment distance of a similar system is given in reference 5. The similar system is the CBU-72/3 weapon which contains 3 warhead parachute combinations (BLU-73A/B) in a single cylindrical casing. Reference 5 lists the filling distance for the aft, center, and foreward warhead chute systems for a number of flight tests. The average filling distance (chute deployment distance) was 45 feet. The standard deviation about this distance was 12.5 feet for the aft bomb, 22 and 23 feet for the center and foreward warheads, respectively. The 12.5 feet standard deviation of the aft bomb is assumed attainable for our system. The standard deviation in deflection and range resulting from variations in deployment distance are

$$\sigma_{cD} = 0 \tag{4a}$$

$$\sigma_{cR} = 12.5 \text{ ft.} \tag{4b}$$

where the c represents chute deployment.

Drift Error: The deployment of the parachute causes the warhead to decelerate rapidly. The warhead then drifts down to impact. The time under chute (from fuze function to impact) varies from 6 to 9 seconds depending on the fuze setting. Random winds during this period can cause a drift error. A steady wind does not cause a drift error, since it affects all the rounds the same way. The random wind is defined as a variation in wind velocity (between trajectories), averaged over an 8 second drift time. A standard deviation in random wind velocity of 1 ft/sec is assumed for this study. This results in a standard deviation in both the range and deflection direction of 8 feet. Therefore, the standard deviations caused by drift in deflection and range are

$$\sigma_{dD} = 8 \text{ ft.} \tag{5a}$$

$$\sigma_{dR} = 8 \text{ ft.} \tag{5b}$$

where the lower case d represents drift. The random wind has equal affect in both the range and deflection directions.

Total Error and Sensitivity: The total error is the sum of the five major components. The total error uses the root-sum-square method to sum the errors. This common method is expressed as

$$\sigma_T = \sqrt{\sigma_\ell^2 + \sigma_r^2 + \sigma_f^2 + \sigma_c^2 + \sigma_d^2} \tag{6}$$

where the subscripts ℓ, r, f, c, and d represent the components due to launcher, rocket, fuze, chute and drift respectively. A few basic concepts of error sensitivity

can be obtained by examining the nature of Eq. 6. For example, assume that each error source contributes 10 mils of error. A 22.4 mil total error results from Eq. 6. If through rigorous tolerances one of the components is reduced to zero, the total error is 20 mils. The total error is still 17.3 mils with two components reduced to zero. The point to be noted in this example is that the sensitivity of each component depends on the other components. The reduction in the total error obtained by reducing the error of a single component may not warrant the increase in cost and/or tolerances required. Consider another case in which one term dominates. For example, assume that one component has an error of 20 mils. To obtain a total error of 20 mils the other 4 components must be reduced to zero. The total error can only be reduced below 20 mils by reducing the 20 mil dominant component. The point to be noted in this example is that a dominant term controls the sensitivity of the whole system. These general points should aid in understanding the sensitivity of this error analysis.

The total error of the components is obtained by using the root-sum-square method as previously indicated. Summing the error contributions, first in deflection, and then in range, given the following expressions

$$\sigma_{TD} = \left\{\left(\frac{5.4R}{1000}\right)^2 + \left(\frac{21R}{1000}\right)^2 + 0 + 0 + (8)^2\right\}^{\frac{1}{2}} \qquad (7a)$$

$$= \left\{\left(\frac{21.6R}{1000}\right)^2 + (8)^2\right\}^{\frac{1}{2}} \text{ ft}$$

$$\sigma_{TR} = \left\{\left(\frac{5.4}{2}\right)^2 \left(\frac{R-570}{1000}\right)^2 + \left[\left(\frac{21}{2}\right)^2 + (12.5)^2\right]\left(\frac{R-570}{1000}\right)^2 + (18)^2 + (12.5)^2 + (8)^2\right\}^{\frac{1}{2}}$$

$$= \left\{\left[(16.5)\left(\frac{R-570}{1000}\right)\right]^2 + (23.3)^2\right\}^{\frac{1}{2}} \text{ ft} \qquad (7b)$$

where the second line of each equation results from summing like terms. These equations indicate two types of terms. Those which are range dependent and those which are constant. The range dependent terms result from angular deviations caused by the launcher and rocket thrust malalignment. Once the rocket is oriented in the wrong direction, the deviation (in feet) continues to increase with range. The constant terms result primarily from deviations in timing (time of fuze operation, time to open the parachute, and average time for the wind gusts). These contributions result in fixed standard deviations which are independent of the range of the vehicle. In general, the range-dependent terms dominate the error at long ranges, while the fixed error terms control the error at the short ranges. For example, consider the total range error (σ_{TR}) expressed in Eq. 7b above. At short ranges ($R < 1500$), the range dependent term is small and σ_{TR} is dominated by the constant term (23.3). For longer ranges ($R > 2000$), the range dependent term begins to dominate. Figure 5 showns the range dependence of the standard deviation for both the deflection error and the range error. Figure 5 indicates that the standard deviation in range and deflection cross at a range of about 1100 feet. The standard deviation at this point is 25 feet. The high slope of the deflection deviation curve indicates that it is dominated throughout by range-dependent terms. The range deviation curve, however, shows the influence of the range-dependent terms at long ranges, but is dominated by the fixed terms at short ranges (low slope). Whether or not the standard deviations can be reduced depends on the sensitivity of the components.

The sensitivity of the components is examined by using a perturbation method. With

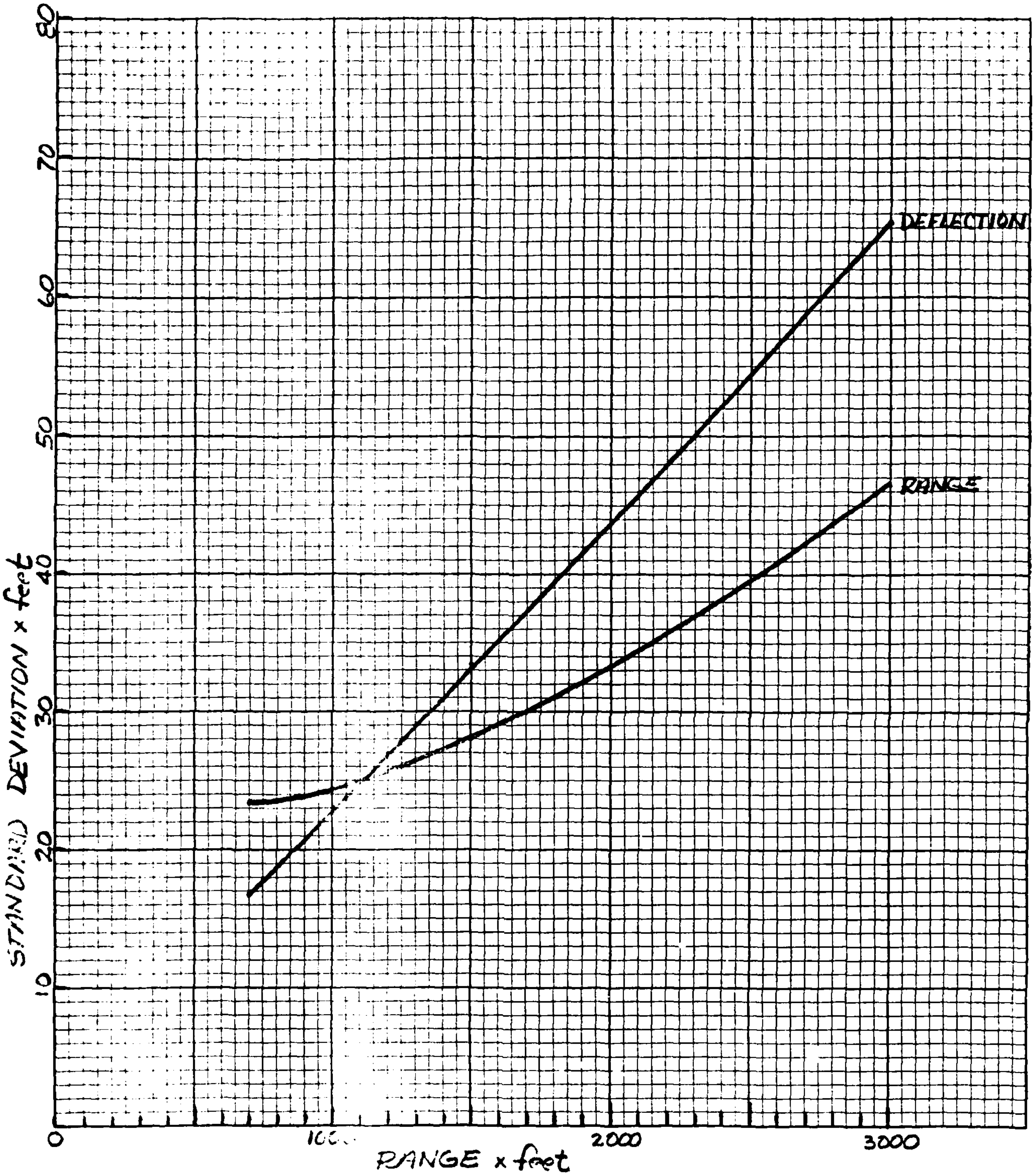


FIG. 5 -- Total Estimated Standard Deviation in Range and Deflection as a Function of Distance Down Range

this method the total deviation error of the sy ᵔ'uated while each component is independently varied. Two conditions (perturbaı. are examined. The first assumes that the component contribution is zero. This variation indicates the potential of reducing the total deviation by reducing that component. The second perturbuation examines the effect on the total deviation error of doubling the component contribution. This variation indicates the sensitivity of the total error to the magnitude of that component. This sensitivity also indicates the criticality of the assumptions previously used in evaluating that component.

Figure 6 shows the sensitivity of the total deflection error to varying each component as a function of range. The dark solid line on Fig. 6 is the total deflection error taken from Fig. 5. This line represents the reference value about which the various perturbations are examined. The figure also presents a list of symbols indicating which components is being perturbed and whether it is reduced to zero or doubled. The deflection error has only 3 error sources; the rocket, the launcher, and the vehicle drift. Among these only the rocket source (thrust malalignment) has a significant affect on the total deflection error. The figure indicates that a considerable reduction in the total deflection error can be obtained by reducing the rocket error component. Doubling the rocket error component essentially doubles the total deflection error. The total deflection error is relatively insensitive to the launcher and drift souces of error. Even doubling these error sources has little affect on the total deflection error.

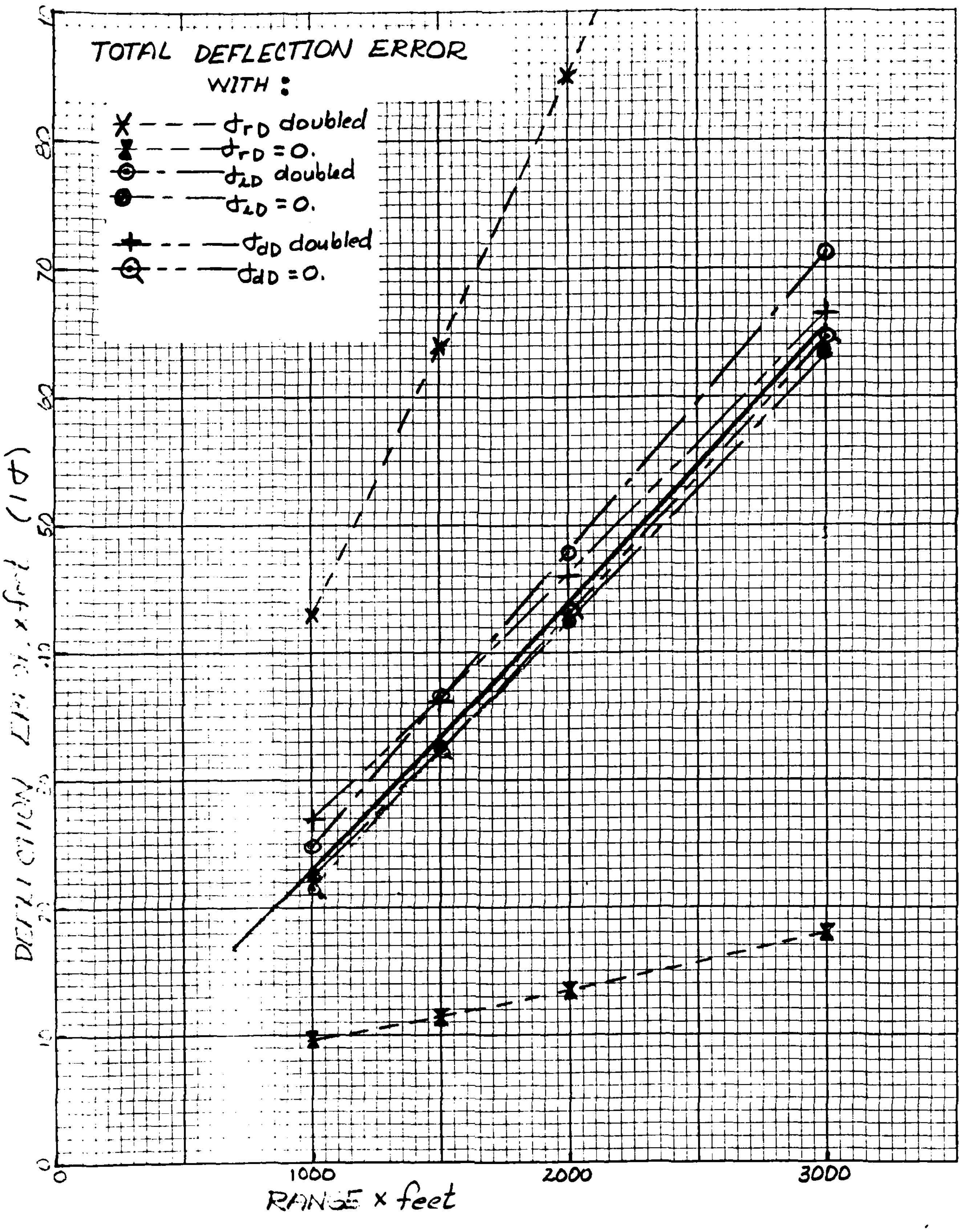


FIG. 6 -- Sensitivity of the Total Error in Deflection to Variations of Each Component Error

The sensitivity of the range error to each component perturbation is shown in Fig. 7 as a function of range. The reference total range error is plotted in the figure as a heavy solid line. Figure 7 presents a list of symbols indicating which component is perturbed. Sensitivity to launcher errors and drift errors is quite small. The launcher and drift errors can be doubled in magnitude without adding much to the total range error. Lines are not plotted through the symbols for these errors as an aid in keeping the figure uncluttered. The total range error is quite sensitive to perturbations in the fuze error. Eliminating the fuze source of error moderately reduces the total range error. On the other hand, doubling the fuze error greatly increases the total range error especially at the shorter ranges. The fuze error being a constant term (independent of range) shows the greatest sensitivity at the shortest ranges. The total range error is moderately sensitive to the chute opening error. The effects of the chute perturbations are about ½ of that of the fuze error component. The total range error is very sensitive to the rocket error component at moderate and long ranges. The rocket range error component consists of two terms. One due to thrust malalignment and the other caused by variations in the total impulse. Both rocket terms are reduced to zero to be consistent with the other component perturbations. Figure 7 also shows the total range error sensitivity to doubling both rocket component terms.

Figure 7 can be a little misleading since the rocket range error component has two

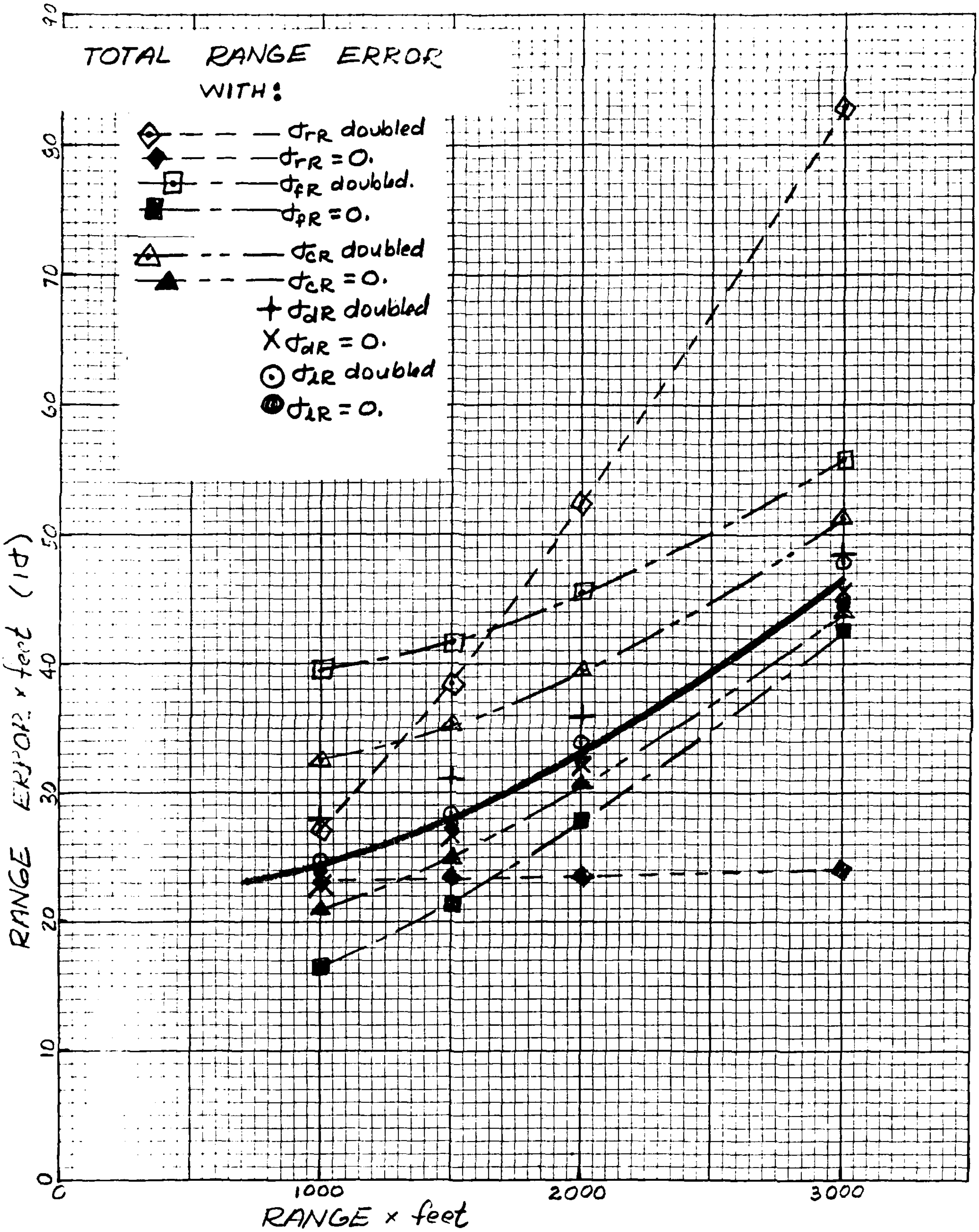


FIG. 7 -- Sensitivity of the Total Range Error to Variations of Each Component Error

terms. Figure 8 shows the sensitivity of the range error to only the rocket error component. The effect on the total range error of having both components of the rocket error equal to zero and both components doubled are replotted on Fig. 8. However, in practice it is difficult to vary both terms in the rocket error. Reducing the mean deviation in the propellant total impulse below 1 percent is difficult and/or costly. Appendix B indicates the thrust malalignment error can be reduced by reducing the burn distance and by adding spin to the vehicle. Figure 8 indicates that reducing only the thrust malalignment error to zero does not greatly reduce the total range error. This low sensitivity results from the large contribution of the total impulse error term. On the other hand, the effect of doubling the thrust malalignment error results in a considerable increase in the vehicle total range error. The reason for this behavior is that both rocket range error terms are about equal. Reducing one only moderately decreases the total. However, doubling one makes it dominant which greatly increases the total.

In general, the total deflection error is sensitive only to the rocket component. The rocket error in deflection results from thrust malalignment of the rocket. Reducing the thrust malalignment greatly decreases the total deflection error. However, reducing the same thrust malalignment error does not greatly affect the range error. This results from the fact that the rocket range error component also has a total impulse variation term. This term must also be reduced to greatly reduce the range error at the longer ranges. The total range error is sensitive at short ranges to the fuze error. A lesser sensitivity is shown at short ranges

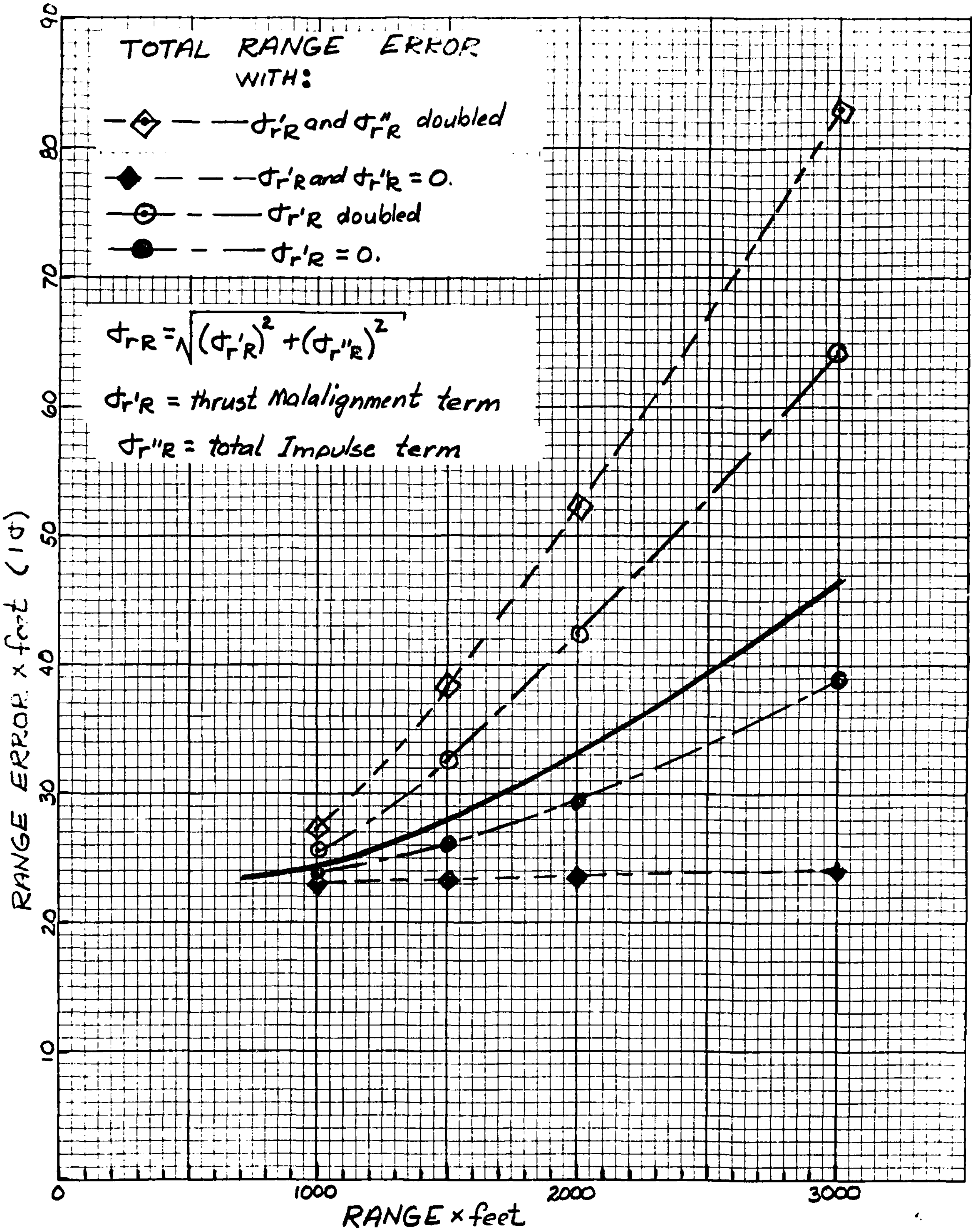


FIG. 8 -- Sensitivity of the Total Error in Range to Variations of the Rocket Component Error

to the chute error. The launcher error and the error due to rocket drift have little affect on either the total deflection or the total range error, even when doubled in magnitude. This result also implies that the assumptions used in evaluating the launcher and drift errors are not critical. The critical assumptions in this analysis are those used to evaluate the thrust malalignment, the propellant impulse variation, the fuze timing deviation, and to a lesser extent, the chute opening variations.

Conclusions: The primary conclusions resulting from this preliminary design and accuracy analysis of a ground launched multiple rocket system are summarized as:

(1) The baseline vehicle is designed similar to the World War II barrage rockets duc to the simplicity and the wealth of available data. This data and design similarity are used to add credibility to the analysis.

(2) The rocket produces a maximum range of 3000 feet. The performance was purposely kept low since the system deviations are directly proportional to the range and the propellant impulse.

(3) The system accuracy indicates the ability of placing one round relative to the proceeding round in a pattern. This accuracy does not have first round aiming errors, but is similar to round dispersion.

(4) The primary sources of system error result from the launcher, the rocket, the fuze timing, the chute deployment, and the round drift due to wind gusts.

(5) Low launch angles decrease both the deflection and range errors. A 30° launch angle is used in the baseline trajectory as a compromise between trajectory range and error contributions (Appendix A).

(6) The sensitivity of the total errors to each error source was examined by a simple perturbation technique. The total deflection error and the total range error are not sensitive to the launcher and the projectile drift error components.

(7) The system total deflection error is sensitive only to the rocket error contribution (thrust malalignment). This source of error can be reduced both by decreasing the burn distance and by spinning the round. A slow spin results in only moderate reductions (Appendix B). A 0.3 second burntime is recommended as a compromise between short burn distances giving smaller rocket errors and the high thrust with potential unfavorable launcher interactions.

(8) The system total range error is sensitive to the rocket error at long range and at short ranges to the fuze timing error and moderately to the chute deployment error. The rocket source of range error is not easily reduced since it results from propellant impulse variations as well as thrust malalignment. The fuze timing error used has a standard deviation of 0.050 seconds.

References:

1. *Ballistic Data: Fin-Stabilized and Spin-Stabilized Rockets. California Institute of Technology, Pasadena, California, 1946.*

2. *Rocket Launchers for Surface Use, California Institute of Technology, Pasadena, California, 1946.*

3. *Davis, Follin, and Blitzer, Exterior Ballistics of Rockets, Princeton, N.J., D. Van Nostrand Company, Inc., 1958, 457 pp.*

4. *California Institute of Technology. Artillery and Aircraft Rockets, by William A. Fowler, Pasadena, California, December 1951, 141 pp (CIT-VM-1).*

5. *Bryant Ferguson, Inflation Characteristics of X-Chute Used with the BLU-73A/B Bomb, Memorandum (Code 4563), Reg. 4563-2621-72 of 19 November 1971, Naval Weapons Center, China Lake, California.*

Appendix A

The Trajectory Effects on the System Accuracy

The vehicle in this study has a rather unique trajectory. The first phase of the trajectory is a standard rocket boost and coast. This phase is followed by the deployment of the parachute which adds a high drag and a rapid turn down of of trajectory. This high drag phase of flight results in a strong moderating effect on the range contribution of error (elevation errors). The deflection error is sensitive to the initial launch angle. This appendix evaluates the interaction of the trajectory on both the range and deflection components of error.

Baseline Trajectory: The baseline trajectory results from minimizing the trajectory influenced errors and meeting the basic system requirements. The baseline trajectory is shown on Fig. 3. The rocket system is launched at an initial QE of 30°. The high thrust motor burns out in ½ second at a range of about 90 feet. The warhead and motor combination then follow a ballistic trajectory indicated by the solid line in Fig. 3. Deployment of the parachute along this trajectory determines the range for warhead impact. Four trajectories with the parachute deployed are shown by the dotted lines in Fig. 3. The chute deployment at ½ second and at 8 seconds represents the minimum and maximum range of the baseline trajectory. The numbers on the baseline trajectory show the time after launch in seconds and the figure also indicates the warhead impact angles.

The trajectory for this sytem has a few operational guideline requirements.

A range out to about 1000 yards was requested. The maximum range of the baseline trajectory (Fig. 3) meets this guideline. In general extending the range of the system increases the errors (as discussed later in this appendix). The time between chute deployment and impact was kept above 5 seconds. This general guideline insures sufficient time for the fuze probe to extend and arm. The warhead effectiveness improves as the impact angle increases. The warhead impact angle was kept above 65^o to meet this general requirement.

An interesting effect on the trajectory results from the rapid deceleration caused by the deployment of the parachute. A feature of this rapid deceleration is essentially a constant range after chute deployment. The added range following parachute deployment is essentially independent of where along the trajectory the chute is deployed. This effect is illustrated in Fig. 9.

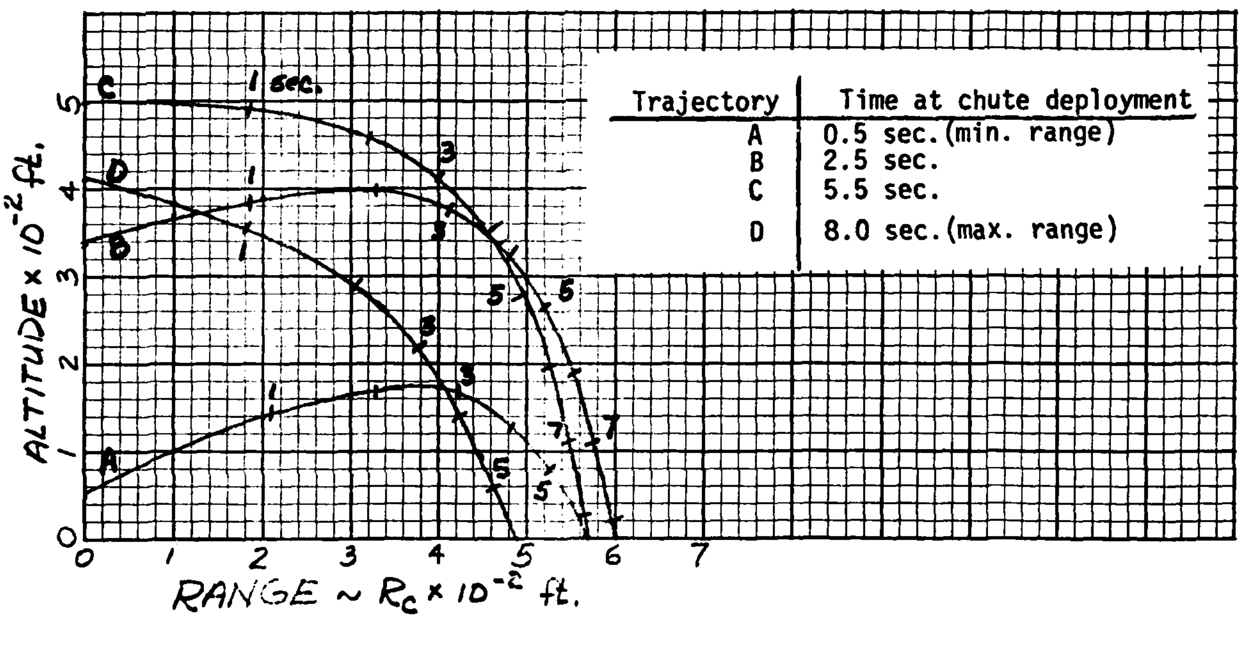


FIG. 9 -- The Phase of the Trajectory with Parachute Deployed

Figure 9 is a replot of the four parachute deployed trajectories from the baseline trajectory. The chute deployed trajectories A, B, and C with deployment times of ½, 2½ and 5½ seconds have ranges (R_C) which vary by only 30 to 40 feet (6%). The maximum range trajectory (D) with a chute deployment at 8 seconds deviates in range from trajectories A and C by about 80 feet (14%)[1]. The use of a constant value for the range after chute deployment simplifies the analysis and allows the equations to be written in a general form. The range under chute deployment used for this vehicle and trajectory is 570 feet. The overall range can therefore be written in terms of the range of the rocket (R_r) and the range after deployment (R_c) as

$$R = R_r + R_c = R_r + 570 \text{ ft} \tag{8}$$

The essentially constant range after chute deployment results from the high deceleration of the vehicle. The first few seconds after chute deployment, as the vehicle progresses down range, result in the largest amount of range. Figure 9 indicates that during the first 3 seconds after chute deployment the vehicle travels 400 of the total 570 feet (70%). Following this period the vehicle turns down rapidly due to its low velocity. In Fig. 9 trajectory B and C acquire 30% more range for an additional 5½ seconds of flight. Eliminating the last 2½ seconds of flight for these trajectories results in a range decrease of only 40 feet. The last 2½ seconds of flight for these trajectories result in an altitude variation of 200 feet. Varying the altitude of the trajectories by 50 feet (10%

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[1] The assumption of constant range after chute deployment is rather strained to meet the 14% (80 feet) range variation of the maximum overall range trajectory D. However, as noted later the use of this constant range assumption is to relate the rocket range R_r to the total range R. The 80 foot deviation is related to ranges greater than 2400 feet and the associated error is less than 3½%.

of the maximum altitude) causes a very small variation in range. This feature of the trajectory is very important in evaluating the pitch or elevation variations in the range error component.

<u>Deflection Error:</u> The deflection error is the error component to the right or to the left of the desired range direction. In this analysis the primary source of deflection error results from thrust malalignment of the rocket. A secondary source comes from launcher orientation errors.

The effect of these error sources is to give the rocket an angular deflection from the desired flight path (ϕ_{fp}). The angular deflection of interest is relative to the ground or target plane (ϕ_g). The angles can be related by means of the launch angle[2]. Figure 10 is a sketch of the relation between the vehicle angular

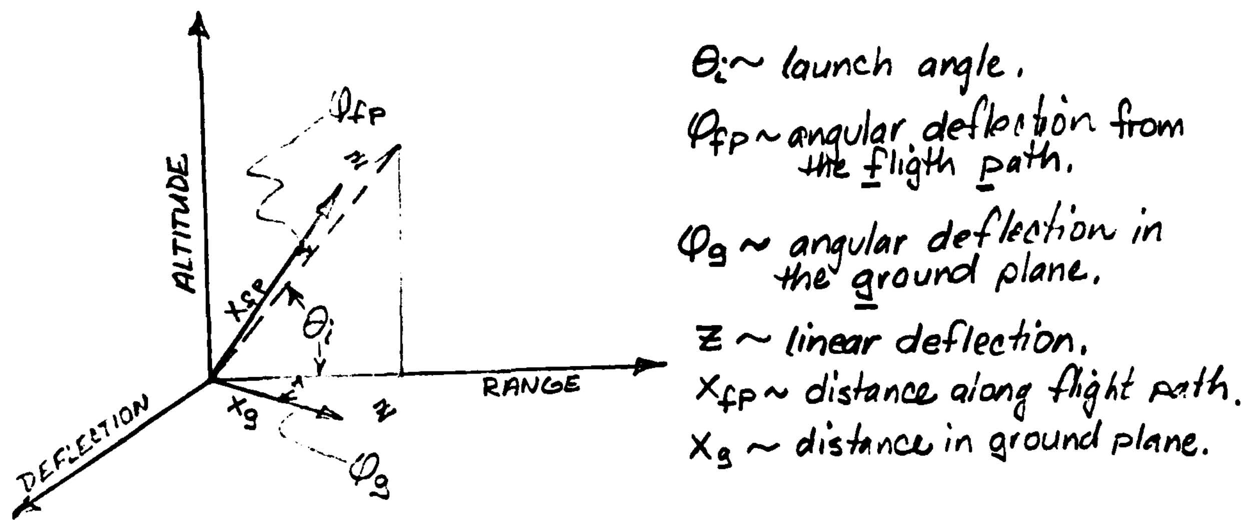

FIG. 10 -- Sketch of the Relation Between the Vehicle Angular Deflection in Yaw and the Launch Angle

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[2]Relating the deflection angles by means of the launch angle assumes that the deflection takes place immediately after launch where the trajectory is relatively straight. This condition is essentially true for launcher orientation and rocket thrust malalignment errors. For cases where considerable tip off and trajectory curvature exists, an averaged θ should be used.

deflection in the yaw plane and the launch angle. The dashed line in the figure represents the desired launch direction. The actual flight direction is indicated by the vector whose magnitude is X_{fp} and whose orientation is deflected ϕ_{fp} from the intended direction. These relations are then projected down into the ground plane. Assuming the deflection angles are small, they can be written in terms of the linear distances as

$$\phi_g = Z/X_g$$

and

$$\phi_{fp} = Z/X_{fp}.$$

The linear distance along the flight path (X_{fp}) can be related to the distance in the ground plane (Xg) by means of the launch angle as

$$\cos \theta_i = X_g/X_{fp}.$$

Combining these equations and solving for the deflection in the ground plane (ϕ_g) gives

$$\phi_g = \phi_{fp}/\cos \theta_i \qquad (9)$$

Equation 9 indicates for the same deflection in the flight path that the deflection angle in the ground plane increases with increasing launch angle. For a launch angle of 30^o, the increase in deflection angle in the ground plane is about

15% greater than the angle in the plane of the flight path. For a 45^{o} launch angle, the ground deflection angle increases by about 41%. Since the deflection angle of the rocket ϕ_{FP} is independent of the launcher angle, Eq. 9 indicates that the launch angle should be kept low. However, reducing the launch angle also decreases the range. The lower launch angles need an increase in the propellant impulse and a higher burnout velocity to travel the required maximum range. Increases in the propellant impulse and the burnout velocity increase the errors in other terms. Simple ballistic theory indicates that the sine of two times the launch angle ($\sin 2\theta_i$) is proportional to the ratio of the range at that launch angle (θ_i) to the maximum range (R/R_{max}). Therefore, the range in terms of the launch angle can be represented as $R = R_{max}\sin(2\theta_i)$. A 30^{o} initial launch angle was chosen as a compromise value for the baseline trajectory. This launch angle gives a range of approximately 85% of the maximum while increasing the flight path deflection angle by only 15%.

<u>Range Error</u>. The range error is the deviation in the range direction caused by system errors. The range error consists of two types of terms. The first type of term depends on the range and results from angular deviations in the pitch plane and variations in the propellant total impulse. The angular deviations result primarily from launcher pitch orientation and rocket thrust malalignment. The second type of range error term is essentially a constant (independent of range). This term results primarily from timing errors and has a strong dependence on the average velocity of the projectile. The major source in this type of error comes from deviations in the fuze timing used to deploy the parachute. Both of these types of error interact with the trajectory parameters in order to produce a range error on the ground. These interactions along with any assumptions made are discussed below.

The range error caused by angular deviations in the pitch plane is strongly influenced by the high drag phase of the chute deployed trajectory. The termination of the rocket phase of the trajectory at a fixed time (by fuzing) also affects the resulting range error. This trajectory interaction can be understood by using the sketch in Fig. 11. In this figure the dashed lines represent the desired vehicle flight path. An angular deviation in the pitch plane above and below the desired flight path trajectory is indicated in the figure. The rocket part of the trajectory terminates at a preset fuze time. Figure 11 indicates the relative postion of the vehicle in space with and without an angular deviation error.

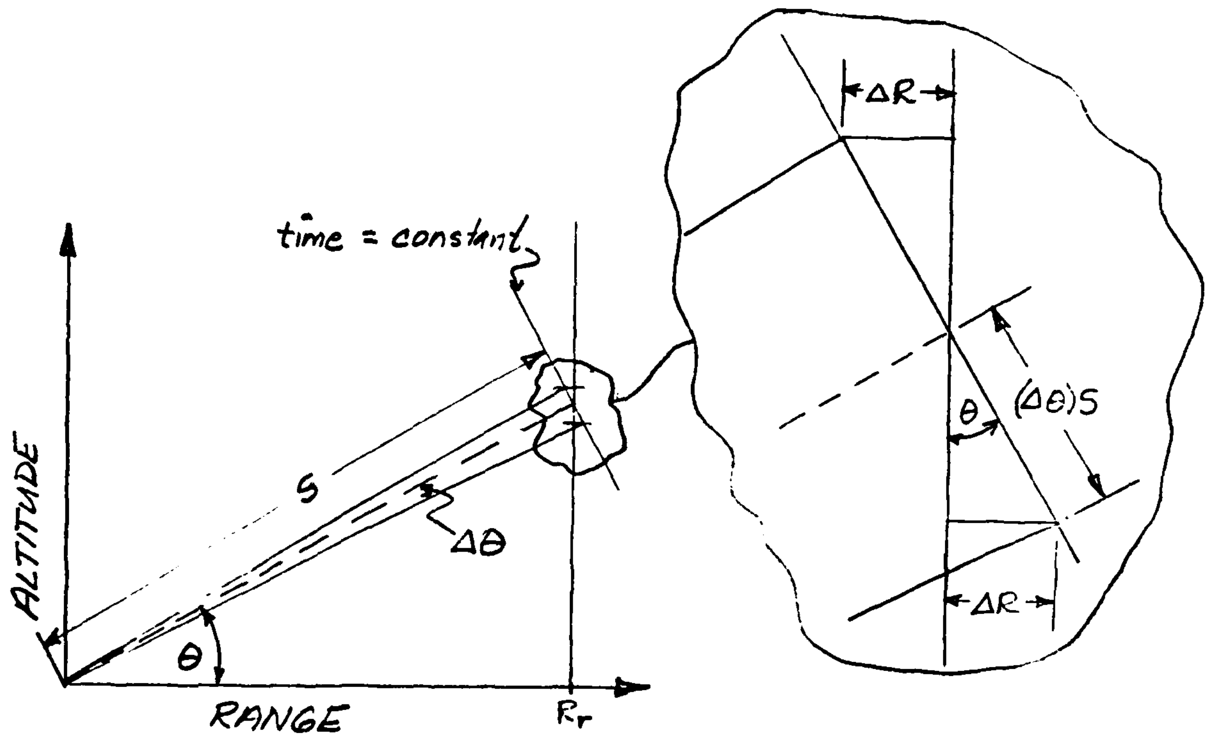

FIG. 11 -- Sketch of a Relation Between the Vehicle Angular Deviation in Pitch and the Launch Angle for a Constant Time

As previously indicated the difference in altitude does not add to the range error. Since the remaining part of the trajectory has a constant range, the range error is evaluated at the end of the rocket phase of flight (R_r). Assuming that the vehicle travels the same distance (S) in the same time, the range error can be written in terms of the angular error as

$$\Delta R = S\,(\Delta\theta)\,(\sin\theta) = \tfrac{1}{2}\,S\,(\Delta\theta) \qquad (10)$$

where the angular deviation error, $\Delta\theta$, is in radians and the launch angle, θ, is set equal to 30°. Equation 10 indicates that the range error (ΔR) is proportional to the sine of the launch angle. Keeping the launch angle low aids in maintaining a low range error. Equation 10 also indicates that the range error component is directly related to the distance traveled (S). At long ranges with a low launch angle the flight path distance (S) can be represented by the range of the rocket[3] (R_r). At short ranges setting S equal to R_r represents a maximum error of 15% for a 30° launch angle. However, at short ranges the error component due to angular deviation in pitch ($\Delta\theta$) is small compared to the range error component from other sources. Replacing S by R_r and substituting the total range for the rocket range from Eq. 8 gives

$$\Delta R = \Delta\theta/2\left(\frac{R - 570}{1000}\right)\text{ft} \qquad (11)$$

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[3]*The use of R_r in place of S at long ranges is a close approximation to the actual situation. The lower flight path for the same time of flight has more curvature and does not project as far down range (ΔR). The effect is to decrease the effective angle θ (Fig. 11). By maintaining θ equal to the launch angle, the shorter R_r in place of S helps to correct for this curvature effect.*

where the division by 1000 converts $\Delta\theta$ from radians to mils (milliradians).

The range error component caused by timing deviations are directly related to the range component of the trajectory velocity. The range component of velocity is equal to the velocity vector in the range direction ($V_R = V\cos\theta$). For a zero drag trajectory, the velocity in the range direction is a constant after burnout. In our baseline trajectory the range component of the velocity varies from 360 ft/sec at rocket burnout to about 275 ft/sec at the last chute deployment point, 8 seconds into the flight. The error in range caused by timing is equal to the product of the deviation in timing and the range component of the velocity $\Delta R = \Delta t(V_R)$. This type of term can dominate the overall range error at short ranges but is relatively unimportant at long ranges where the range dependent terms dominate. Therefore a constant range velocity component of 360 ft/sec is assumed in this analysis. This greatly simplified the general expression of the equations.

Appendix B

Rocket Accuracy Analysis and Tradeoff

In this appendix the errors resulting from the propulsion phase of flight are evaluated. These rocket errors can greatly affect the system accuracy. The accuracy analysis in this report indicates the rocket source of errors dominate the overall deflection error at all ranges. The rocket errors also dominate the total range error at the longer ranges. The purpose of this appendix is to evaluate and understand the source and cause of the rocket errors, and to indicate what can be done to reduce these errors.

Rocket Error Sources: *The rocket by definition receives an initial velocity by means of a high initial thrust and then follows essentially a ballistic trajectory. The magnitude of the boost can vary by means of propellant total impulse. The direction during boost can vary when the rocket thrust is off axis. The ballistic phase of the trajectory can vary by unwanted aerodynamic forces. These variations in total impulse, thrust direction, and aerodynamic forces represent the three major causes of rocket error.*

Rocket propellant total impulse variations are a major source of range errors. The propellant total impulse is the integral of the thrust over the burntime. Variations in the impulse essentially vary the projectile velocity and hence, range. A primary source of propellant impulse variations results from differences in the propellant weight. This weight variation can be quite large when comparing lot to processing lot and different extrusions. (One would expect that motors in the same group of rockets would come from the same batch.) Other sources of propellant impulse variations results from high burn rates and grain temperatures. High thrust

designs can result in unburned propellant being expelled through the nozzle of the motor. This effect causes a variation in the propellant impulse. The burn rate increases as the grain temperature increases. This can affect the propellant impulse in the same way as high burn rates[4].

Thrust malalignment causes the projectile to rotate away from the intended direction of motion. This change in vehicle orientation produces a system error in both the deflection and range directions. The primary source of thrust malalignment results from mechanical and gas malalignments. The mechanical malalignment results from fabrication tolerances, projectile asymmetries, and off-axis nozzle alignment. These inaccuracies result in the thrust nozzle axis not passing through the vehicle center of gravity (cg). The gas malalignment results from disturbances in the exhaust flow which deflects the thrust axis from the nozzle axis. Gas malalignment was identified by C.I.T. personnel as a pertinent source of error (reference 4). High chamber Mach numbers and the nature of the burning grain surface can aggravate this source of thrust malalignment.

Aerodynamic interactions represent the third major source of rocket errors. Aerodynamic errors result from malalignment and instabilities. Asymmetries and malalignment in manufacture can produce unwanted aerodynamic forces. For example , a fin malalignment can cause the vehicle to spin and deviate from the basic ballistic trajectory. Barrage rocket manufacturing during World War II

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[4] Varying the burn rate affects the burntime and distance. As indicated later this effect can greatly influence the error produced by thrust malalignment.

indicated that the aerodynamic malalignments can be kept quite small (references 3 & 4). The resulting error caused by these aerodynamic sources is small in relation to the error from the other sources (thrust malalignment and total impulse variations). Careful projectile manufacture and assembly should eliminate this aerodynamic source of rocket error from further consideration.

Projectile instability is sometimes mentioned as a source of aerodynamically produced rocket error. While the basic fin stability of the vehicle must be assumed, some roll-yaw instability can occur. The projectile has a natural yaw oscillation frequency which depends on velocity. If the vehicle has a spin or roll rate equal to the natural yaw frequency, coupling can occur. This coupling results in a coning motion which is basically unstable and leads to trajectory errors. Essentially this analysis assumes that roll/yaw coupling does not occur. The estimated yaw oscillation frequency of the baseline vehicle is about 1.5 revolutions per second (RPS).

<u>Total Impulse Variations:</u> A major cause of rocket error results from variations in the propellant total impulse. Such variations result in errors in range. Since the variation of impulse is in the range direction there is no direct error contribution in deflection. The range error arises from the deviation of the velocity of individual rounds from the mean, due primarily to the variations in the amount of propellant burnt effectively in the motor (total impulse). A variation in projectile inert weight also affects the vehicle velocity. Reference 4 indicates that the barrage rockets developed during the Second World War exhibited mean deviations in

velocity from 0.75% to 1.25% (the mean deviation is approximately 0.8 of the standard deviation). For this analysis a 1% mean deviation (1.25% standard deviation) is assumed for the baseline vehicle.

The range of a standard ballistic rocket is essentially proportional to the velocity squared. The uniqueness of this trajectory (Appendix A) results in the range being dependent on the velocity to the first power. The linear dependence on velocity results from the rocket range being determined by a fixed fuze time. The range of the rocket can be written as

$$R_r = \bar{V}t \approx C_1 V_b t \approx \frac{C_1 I_T t}{M}$$

where C_1 is a proportionality constant and $V_b \approx I_T/M$ is the burnout velocity in terms of the total impulse (I_T). The deviation in range is obtained by the differentiating this equation to obtain

$$\frac{\delta Rr}{R_r} = \frac{\delta I_T}{I_T} + \frac{\delta t}{t} - \frac{\delta M}{M} \approx \frac{\delta I_T}{I_T} \qquad (12)$$

where the deviation in time is zero due to the fixed fuze timing, and the deviation in mass is very small[5]. Equation 12 indicates that the percent error in rocket range is the same as the percent error in total impulse[6]. A rocket range error

[5]The δM term includes only the variation in mass resulting from the difference in propellant weight. Variations in the total mass of the round are included in the total impulse variations by the assumption that the variation in total impulse causes the differences in the vehicle velocity.

[6]The maximum range error of a standard ballistic projectile is approximately two times the percent error in vehicle velocity. The effect of the constant fuze timing greatly moderates the error in range resulting from differences in the velocity of the vehicle.

of 1.25% or 12.5 mils results from assuming 1.25% standard deviation in the total impulse.

<u>Thrust Malalignment:</u> Thrust malalignment results when the thrust axis does not pass through the center of gravity of the vehicle. Upon leaving the restraint of the launcher, this malalignment causes the vehicle to rotate about its transverse axis. The rotation coupled with the thrust results in an angular deviation from the intended direction. Since the malalignment can be in any direction, the resulting deviation can be in the range and/or deflection direction. Figure 12 is a sketch of the rocket vehicle showing the effects of thrust malalignment.

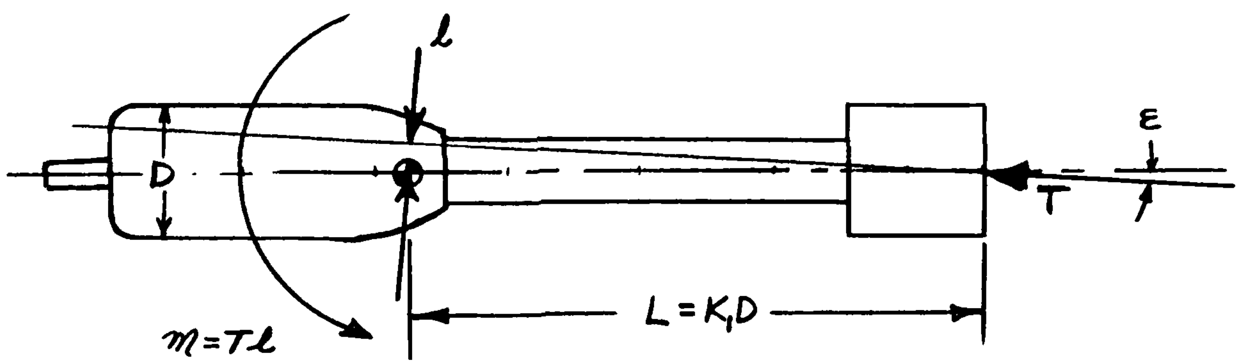

FIG. 12 -- Sketch of the Rocket Thrust Malalignment

The figure sums all the various sources of thrust malalignment and presents them as a thrust axis angular deviation (ε). The malalignment of the total thrust (T) from the center of gravity by the amount ℓ produces a rotation about the transverse

axis. Using this torque ($T\ell$) as the only moment, the angular position (ϕ) at any time can be written as

$$\phi = \iint \left(\frac{T\ell}{I}\, dt \right) dt = \frac{T\ell}{2I}\, t^2 = \frac{T\ell}{2MK^2}\, t^2 = \frac{G\ell}{2K^2}\, t^2$$

where $I = MK^2$ is the moment of inertia of the rocket about the transverse axis, K is the radius of gyration, and $G \approx T/M$ is the forward acceleration of the rocket. The lateral component of acceleration caused by the rocket rotated off the initial direction by ϕ is $G\sin\phi \approx G\phi$. Integrating the lateral acceleration from $t = 0$ to burnout $t = t_b$ gives the lateral velocity at burnout as

$$v_L = \int_0^{t_b} G\phi \; dt = \frac{G^2\ell}{6K^2}\, t_b^{\,3}$$

while the forward velocity is

$$v_b = G\cos\phi \; t_b \approx G t_b \quad .$$

The resultant direction of motion at burnout becomes

$$\theta_b \approx \tan\theta_b = v_L / v_b = \left(G\ell/6K^2 \right) t_b^{\,2} = 1/3(d_b\,\ell/K^2) \qquad \text{(Radians)}$$

where $d_b \approx \frac{1}{2} G t_b^{\,2}$ is the distance traveled during burning. This simplified derivation assumes a zero length launcher and no aerodynamic moments. The launcher

essentially constrains the rotation of the round until the rocket leaves the launcher at a time equal to t_p. Integrating the equations from t_p to t_b gives the somewhat more involved expressions

$$\phi_b = (G\ell/2K^2)\,(t_b - t_p)^2 \qquad (13)$$

$$\theta_b = 1/3\,(d_b\ell/K^2)\left[1 - \sqrt{P/d_b}\right]^3 \qquad (14)$$

where p is the effective launcher length. Equation 13 expresses the angle at burnout that the vehicle has rotated about the transverse axis. Equation 14 expresses the change in heading direction at burnout caused by the malalignment. Therefore, Eq. 13 is the sum of the heading direction plus the yaw angle that the vehicle has at burnout. Equation 14 shows that the angular deviation is inversely proportional to the transverse radius of gyration (K). This relationship results since the transverse moment of inertia resists the rotation produced by the thrust malalignment[7]. The angular deviation is directly related to the distance ℓ that the thrust passes the cg. This direct dependence occurs since the torque ($T\ell$) is the moment producing the rotational error. Equation 14 also shows the direct dependence of the angular error on the vehicle burn distance (d_b). For burn distances within the

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[7]The rocket design has a relatively high ratio of motor length to motor diameter ($\sim$ 10). The long motor length maintains a high transverse radius of gyration and transverse moment of inertia.

launcher, the angular deviation is essentially zero. Maintaining short burn distances results in relatively low angular deviations. The simplified expression for the angular deviation (Eq. 14) was derived assuming zero aerodynamic forces (in vacuum). At short burn distances, however, equation 14 should indicate the correct relationship since there has been little time for the aerodynamic forces to act. The range of burn distances for which Eq. 14 is valid must be determined by deriving the angular deviation with the aerodynamic terms included.

Professor I. S. Bowen first derived the general solution of this dispersion problem in 1943. His work is included in both references 3 & 4. However, the general solution of this problem must be expressed in terms of Fresnel integrals. Since Fresnel integrals are somewhat unfamiliar, both references 3 & 4 derive approximate solutions whose behavior with the important variables is more easily recognized. Reference 3 derives two (2) approximate solutions whose region of validity depends on burn distance. These equations are written as

$$\theta_b = 1/3\,(d_b \ell/K^2)\left[1 - \sqrt{P/d_b}\right]^3 \left[1 - \left(d_b/\lambda\right)^2\right] \text{ radians } (d_b \leq .6\lambda) \qquad (15)$$

and

$$\theta_b = 1/8\,(\lambda \ell/K^2)e^{-4\sqrt{P/\lambda}} \text{ radians } (d_b > .6\lambda) \qquad (16)$$

where λ is the yaw oscillation distance. The yaw oscillation distance λ is the

distance traveled by the vehicle while completing one yaw cycle due to the stabilizing action of the fins. For example let the vehicle have zero yaw angle on leaving the launcher. Assume a thrust malalignment rotates the vehicle to the right. As the velocity increases a restoring moment caused by the fins rotates the vehicle back towards zero yaw. The rotational inertia causes the projectile to rotate past zero yaw into a left yaw. The fin produced moment again opposes the yaw and drives the vehicle back to the zero yaw position. At this point the vehicle has completed one yaw cycle. Inertia continues the cyclic yaw motion but damping decreases the yaw amplitude. The yaw oscillation distance depends on the transverse moment of inertia of the vehicle and the aerodynamic restoring moment generated by the fins. However, to a first approximation, the yaw oscillation distance is independent of the vehicle velocity. With the thrust malalignment driving the vehicle into an angle of yaw, the yaw oscillation distance is a basic parameter in correlating the angular deviation.

The approximate solution for the angular deviation at short burn ranges ($d_b \leq .6\lambda$) *is given by Eq. 15. This equation is very similar to the simply derived Eq. 14 which assumed no aerodynamic forces. The difference arises in the addition of one term to take into account the effects of the fin action of the vehicle. As the burn distance decreases, the term approaches zero, since there is insufficient time for the fins to act.*

Once past a certain burn distance the fins dominate the rotation and one would expect the yaw oscillation distance to replace the burn distance as the controlling

parameter. Intuitively one might expect the critical yaw oscillation distance to range between ¼ and ½ of the yaw oscillation cycle. These fractions of the yaw oscillation cycle represent the phase where the vehicle rotates back to zero yaw from the maximum yaw angle at ¼ cycle. Distances greater than this critical part of the yaw oscillation distance would not add to the angular deviation. That is beyond this critical distance, the deviation caused by thrust malalignment is cancelled as the vehicle yaws in the other direction. This critical distance was determined from the exact solution (reference 3) to be 3/8 of a yaw oscillation cycle (½ way between ¼λ and ½λ). The effect of the fin action on the angular deviation can be estimated by replacing the burn distance d_b by the critical fin control distance (3/8λ). Making this substitution in Eq. 15 results in an equation similar to Eq. 16[8]*. The exponential term in Eq. 16 is the correction for the launch length.*

The thrust miss distance ℓ is the only term in Eq. 14, 15 & 16 which cannot be calculated from a preliminary vehicle design. The other terms are easily calculated or measured from simple rocket tests. The thrust miss distance ℓ can be statistically obtained for a specific vehicle by testing a large number of rockets. For example, reference 4 indicates the 4.5 inch Mk1 Mod 0 barrage rocket has a mean linear malalignment of .012 inches due to mechanical malalignment and .026 inches due to gas malalignment . Assuming these sources are random and summing gives a standard deviation of 0.023 inches or 0.0019 feet. To evaluate the angular deviation of the baseline vehicle, a method for estimating the linear malalignment must be derived.

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[8] Actually, the burn distance in the cubic term is replaced by 9/16 of the yaw oscillation distance. The cubic term is then expanded and the expansion is represented by the exponential term shown in Eq. 16.

Consider again the sketch of the rocket vehicle showing the effects of thrust malalignment in Fig. 12. The thrust axis deviates from the axis through the vehicle cg by an angle ε. The angle ε is small, on the order of 1/10 of one degree (6 minutes of arc) for the 4.5 inch diameter round. This thrust malalignment angle would be expected to vary with changes in the rocket design. However, for similar designs the angle ε is assumed to be independent of the rocket diameter. Therefore, the linear malalignment ℓ can be written in terms of the diameter as

$$\ell = L \tan \varepsilon \quad K_1 D \varepsilon = K_2 D = .000422\, D \qquad \text{feet} \qquad (17)$$

where the constant $K_2 = K_1 \varepsilon = .000422$ ft/in using the values for the 4.5 inch diameter Mk1 barrage rocket. Equation 17 indicates that the linear malalignment is only a function of the diameter. To check this equation, the linear malalignment is estimated for vehicles of different diameters. These values are then used to calculate the lateral deviation of known barrage rockets. Such a correlation between the calculated lateral deviation and the observed test results is shown in the table in Fig. 13. The calculated lateral deviations were evaluated using Eq. 15 & 17. The needed terms in the equations came from test data (ref. 1). The round listed in Fig. 13 were chosen because sufficient flight tests during the 1940's allowed observed lateral deviations to be plotted in Ref. 1 as a function of grain temperature. As noted in the figure, the propellant burntime is a strong function of the propellant grain temperature. Firings in the winter when the rounds and their propellant grains are cold have a much longer burntime, and hence, a greater lateral deviation than tests in hot climates[9]. The grain temperature varies from 15°F to 120°F. Variations in burntime for this grain temperature change is also indicated in the

[9] The extremely large variation in burntime with grain temperature shown for these rounds results from the motor design. These extruded grains are designed to burn on both the inside and the outside of the grain. This allows the temperature of the wall to influence the burn rate. Grains designed for internal burning only exhibit much less sensitivity to grain temperature.

ROUND	V_b ~ burnout velocity ft/sec	ℓ ~ malalignment[b] ft.	grain temperature °F	t_b ~ burn time sec.	d_b ~ burn distance ft.	θ_b ~ lateral deviation[c] CALCULATED mils	θ_b ~ lateral deviation TEST mils
4.5 Mk I Mod O	355	.0019	15	.60	106	80	75
	↓	↓	70	.38	67	47	40
			120	.22	39	21	22
4.5 BR (250yd)	150	.0019	80	.19	14	2	6
7.2 Mk7 Mod O	220	.0030	15	.53	58	35	40
	↓	↓	70	.33	36	16	26
			120	.23	25	8	14
7.2 Mk I Mod2 (CIT 7.2 R.M. 17)	165	.0030	15	.60	50	30	17
	↓	↓	70	.36	30	12	11
			120	.23	19	4	7
2.5 in. Dia. Rocket Grenade	360	.0010	15	.20	36	18	13
	↓	↓	70	.13	23	9	8
			120	.10	18	5	4

[a]Values needed for calculations taken from tests (Ref. 1).

[b]The malalignment is assumed to scale with diameter.

[c]Lateral deviation is referenced to the intended flight path.

FIG. 13 -- Correlation Between the Calculated[a] Lateral Deviation (Theory) and Observed Test Results from Ref. 1

figure. The rocket burn distance is directly proportional to the burntime ($d_b \approx \frac{1}{2} V_b t_b$). The last two columns in Fig. 13 compare the calculated lateral deviations with the test values expressed in mils. The correlation of these values is actually better than one would expect from the simplicity of the analysis. The correlation seems to be good at least for the diameters checked (2.5 to 7.2 inches). The 7.2 inch diameter is the largest barrage rocket of this type designed and tested by C.I.T. (reference 1). The good correlation in Fig. 13 is the basis for the extension of this analysis to the 13 5/8 inch diameter baseline vehicle.

The angular deviation for the baseline vehicle can be estimated by using Eq. 15, 16 & 17. Since the rocket impulse determines the vehicle burnout velocity and is held constant, decreasing the burn distance increases the rocket average thrust level. Both the estimated angular deviation and the required thrust level T are shown in Fig. 14 as a function of the burn distance and burntime. The solid line represents the standard angular deviation and the dashed line the average required thrust level. The reference conditions for the baseline vehicle are listed on the figure. A 0.5 second rocket burntime is the baseline value used in this report. These conditions result in a 21.6 mil standard angular deviation and an average thrust level of 5250 lbs. The triangular points indicate the effect of varying the effective launcher length p. The baseline reference effective launcher length is 5 feet. Cutting this length in half (p = 2.5 feet) increases the angular deviation to over 29 mils. Doubling the effective launcher length to 10 feet reduces the angular deviation to about 14.5 mils. Since a short effective length leads to high rocket tip off while a long launcher length means a high launcher weight, the baseline 5 foot effective launcher length represents a good compromise value.

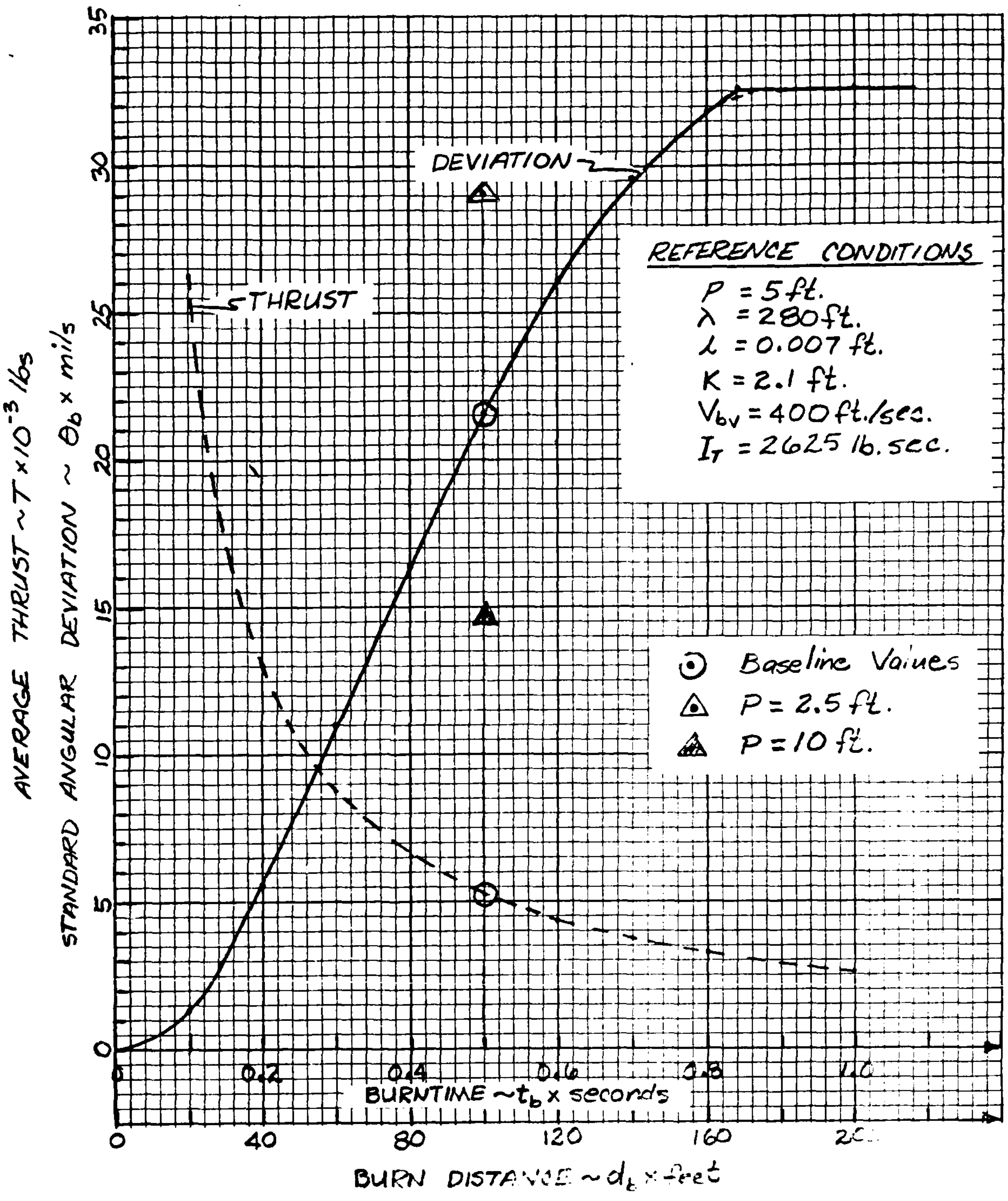

FIG. 14 -- Estimate of the Rocket Thrust Malalignment Deviation

The 0.5 second burntime is used for the baseline vehicle since it is easy to achieve in a 5 inch diameter motor design. A 0.3 second burntime significantly reduces the angular deviation over that of the 0.5 second burntime. This short burntime requires an 8750 lb. average rocket thrust level. This high thrust level can result in significantly increased rocket launcher interactions. Whether the interaction level is acceptable depends on the launcher design and will have to await testing. Modifications to existing motors can produce the 0.5 second baseline burntime. However, if a new grain and motor is developed, serious consideration should be given to designing a 0.3 second burntime. The resulting shorter burn distance could significantly reduce the angular deviation due to thrust malalignment.

Finally, a few words need to be said about producing a slow spin[10] to decrease the deviation induced by the thrust malalignment. Immediately off the launcher a decrease in effective thrust malalignment results by spinning the vehicle. The thrust malalignment direction is rotated around the desired flight path reducing the deviation in any single direction. The benefit derived from inducing a slow spin depends on the projectile requirements and performance. The low velocity of the baseline vehicle results in a low ($\sim$1.5 rps) average yaw oscillation frequency. To prevent yaw/roll instabilities, the spin must be kept above or below this natural yaw frequency. The baseline vehicle requires a parachute deployment and a liquid filled warhead. Both of these can produce problems at even moderate spin rates. Therefore, the spin rate for the baseline vehicle should be low (rps < 5).

[10] Slow spin is a roll rate given to a fin stabilized projectile to aid in nulling malalignments. The magnitude of the spin is usually between one to two orders of magnitude less than the spin required for spin stabilization.

The low spin rates and short burntimes result in only moderate reductions in the angular deviation. As an example consider an average spin rate[11] of 2 revolutions per second and a burntime of 0.3 seconds (desired). During this critical period, the projectile rotates only 0.6 of a revolution. Although some distributions of the thrust malalignment results, the reductions in the angular deviation are relatively small (~ 20%).

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[11] *Usually the spin is produced by the motor using vanes or scallops in the nozzle. The spin starts at zero and increases to burnout. The average spin rate is the mean value over the critical flight distance.*